Evaluating Corporate Bonds and Analyzing Claim Holders' Decisions with Complex Debt Structure

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- A corporate bond is a popular financing or investment tool
  - outstandings in the US market: 2248 billions in 1996
    7846 billions in 2014 (from SIFMA)

- A default-free bond (e.g., Treasury bond) can be evaluated separately from other simultaneously outstanding default-free bonds.

  However, the value of a defaultable bond and the claim holders' (i.e., bond issuer and holders) decisions may be greatly affected by the existence of other outstanding bonds of the same issuer.

  - May be at least due to two effects:
    [1] claim dilution:
    e.g., new bond issuances dilute the values of other previously issued bonds of the same issuer (Fama and Miller, 1972; Ingersoll, 1987)

    [2] wealth transfer:
    e.g., a callable bond is redeemed late due to the existence of other bonds of the same issuer (Longstaff and Tuckman, 1994)

- **To capture these effects, we evaluate corporate securities through structural models based on the issuer's capital structure**

  (e.g., Merton(1974); Leland(1994), etc.)

- **However, extant structure models usually oversimplify the debt structure. That probably makes these models produce unrealistic results**

  (see Rauh and Sufi, 2010; Colla et al., 2013)

  - Actually, a firm relies on various types of bonds simultaneously, including different maturities, priorities and covenants

  - However, existing models use, for example,

    [1] one bond as a representative for the whole debt structure

    (e.g., Merton (1974), Kim et al. (1993) etc.)

    [2] an unified default trigger that has limited relation to the debt structure

    (e.g., Black and Cox (1976), Zhou (2001) etc.)

    [3] the default trigger shaped by the debt structure according to strong assumptions on the issuing firm's future financing policies

    (e.g., Geske (1977), Leland and Toft (1996) etc.)

- **Oversimplifying an issuing firm's debt structure may oversimplify its default triggers (i.e., the default probability) and the corresponding losses of promised payment to each bond holder due to liquidation (i.e., recovery rate)**

  - **That may be why extant structural models reach only limited success empirically.**

    (see Eom et al.,2004; Davydanko, 2012; Huang and Huang, 2012)

- **To promote the empirical validity of structural models**

  - **choose the compound option approach rather than the portfolio of zeros approach (Eom et al., 2004)**

- **Given the compound option approach, we incorporate two novel concepts when developing our structural model**

  - a debt-structure-dependent default trigger
  (respond to Davydenko(2012))

  [1] characterize an issuing firm's bond repayment schedule and the corresponding amounts, and the amounts of its asset frozen by restrictive bond covenants

  [2] capture potential changes of the repayment schedule driven by call or put provisions

  - measuring the firm's insolvency risk through its remaining assets

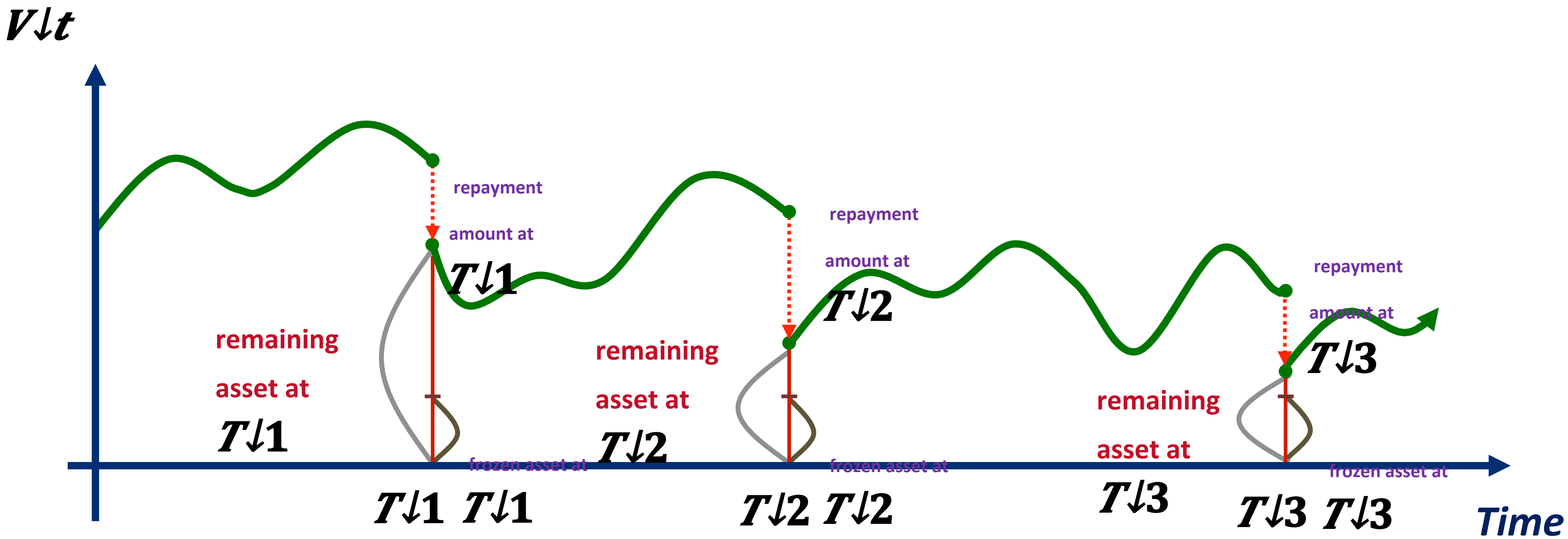
  [1] previous studies exogenously specify one or two repaying policies, like, total equity financing (e.g., Geske (1977)) or stationary debt structure (e.g., Leland and Toft (1996))

  [2] with the concept of remaining assets, we do not assume financing policies
  - remaining assets is a unbiased proxy for the level of internal funds and ability to raise external funds

- An issuing firm's repayment schedule and its insolvency risk

  - Denote the market value of the firm's asset by V_t

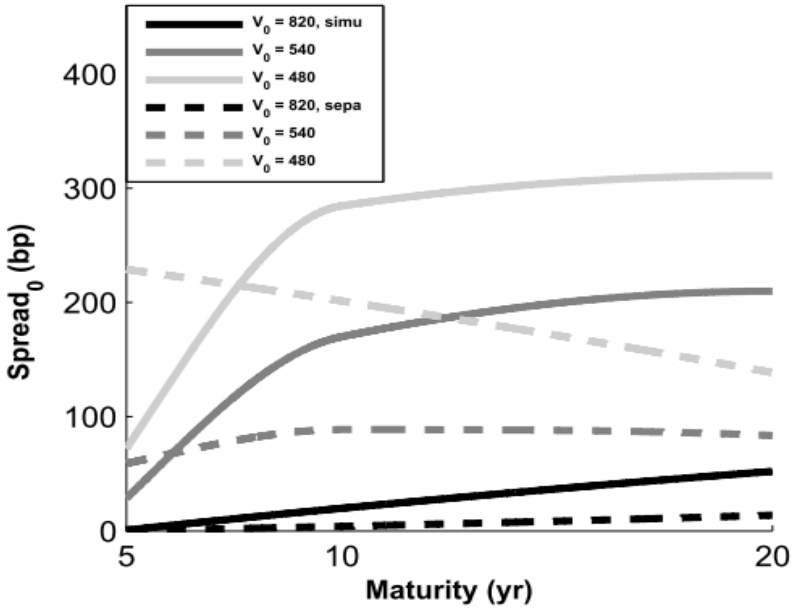
  - The firm's debt structure is composed of three bonds B_1, B_2, and B_3 with time to maturity T_1, T_2, and T_3. Let $T_1 < T_2 < T_3$



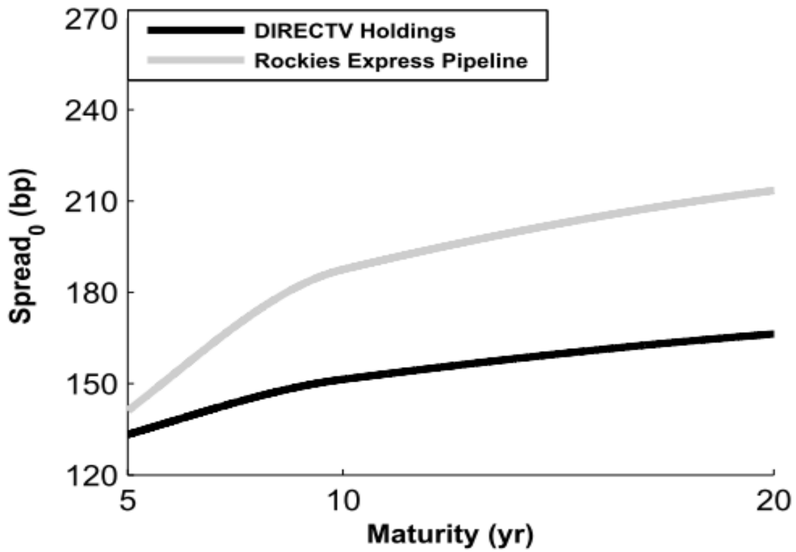
- **Our framework generates reasonable quantitative results consistent with empirical observations** **(respond to Eom et al. (2004))**

  - For example, when evaluating the otherwise identical bonds with different maturities and drawing the corresponding yield spread curves through our model (i.e., bonds are simultaneously priced) and the otherwise identical model using (1) one bond as a representative for the whole debt structure (i.e., bonds are separately priced)

solid curve: by our model
dash curve: by the model with (1)



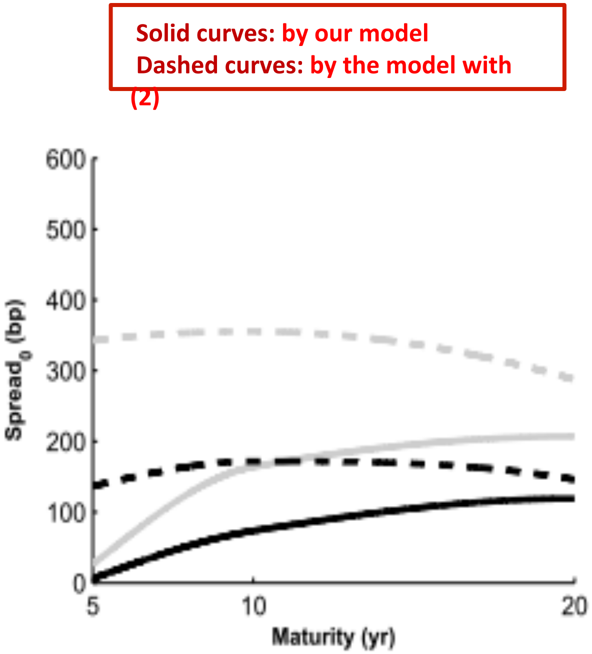
real world observations

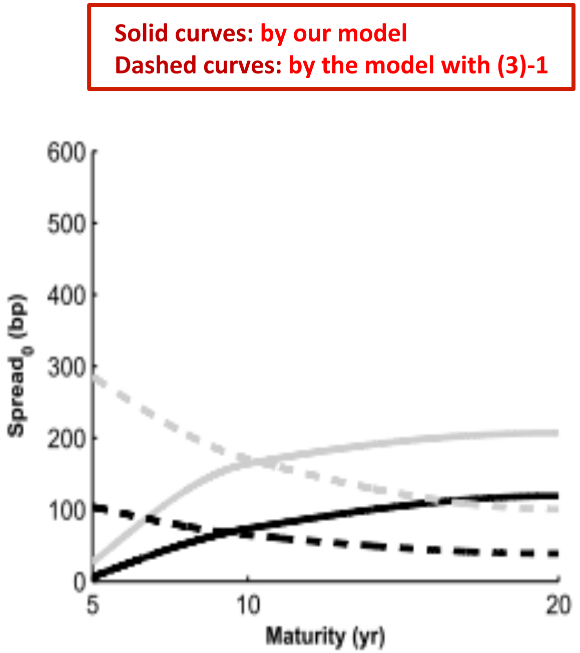


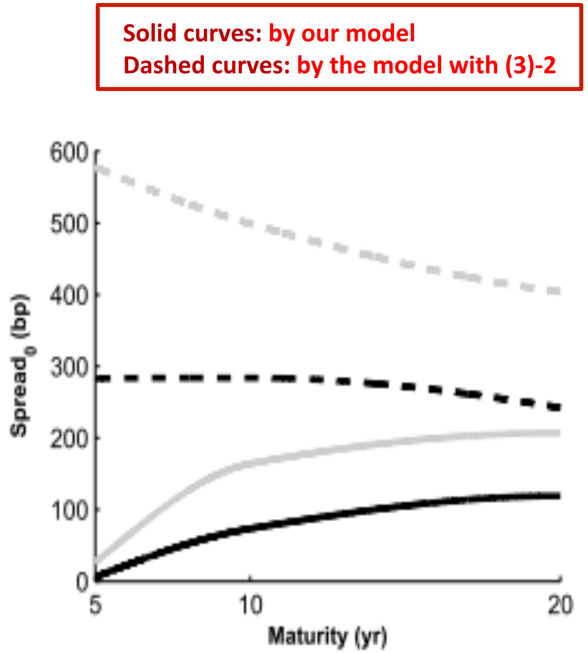
(Helwege and Turner, 1999 ; Huang and Huang, 2008)

- through our model and the otherwise identical model **(2)** with an unified default boundary specified exogenously without consider issuing firm's payment schedule

- through our model and the model with the default boundary shaped by the debt structure according to strong assumptions about the issuing firm's refinancing policies, like **(3)-1**: total equity financing and **(3)-2**: stationary debt structure





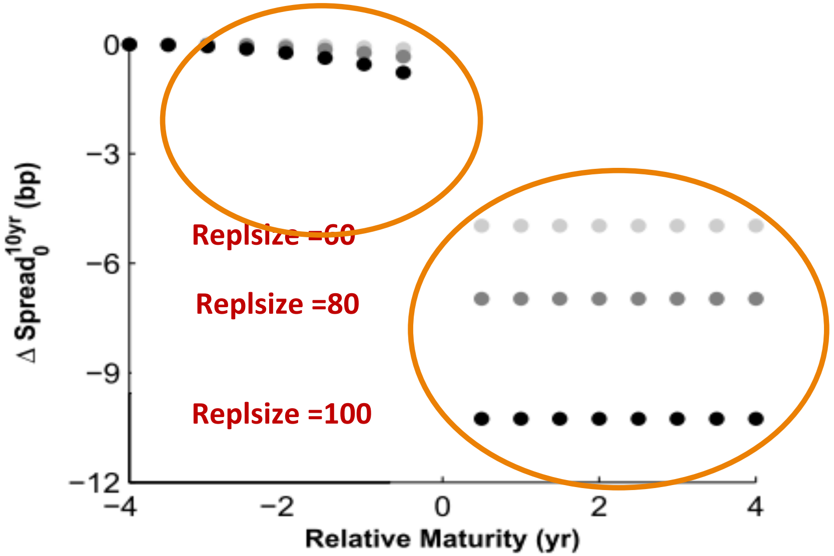


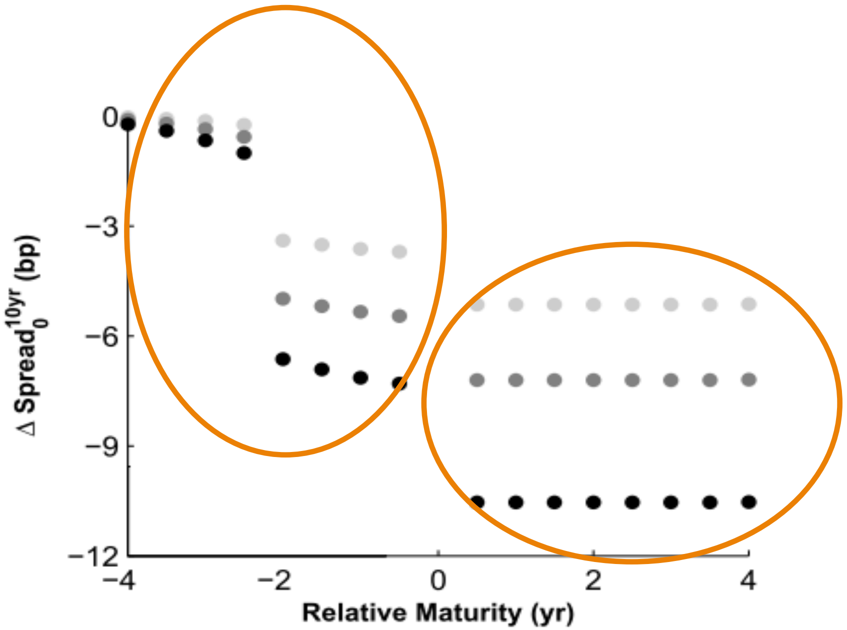
- **The holders of the previously issued senior bonds have the right to block the scheduled payments occurred within the certain period to the newly issued junior bonds once their payments are not fulfilled.** **Davydenko (2007)**

  - **the yield spread of an existing senior bond would decrease once its seniority is improved (e.g., as an existing bank loan is replaced by a new junior bond)**
    (Linn and Stock, 2005)

  - **Ingersoll (1987) predicts that it would make significant difference between the case that the firm raises short-term junior bonds and that raises the long-term.**

    However, Linn and Stock (2005) cannot significantly observe that difference empirically

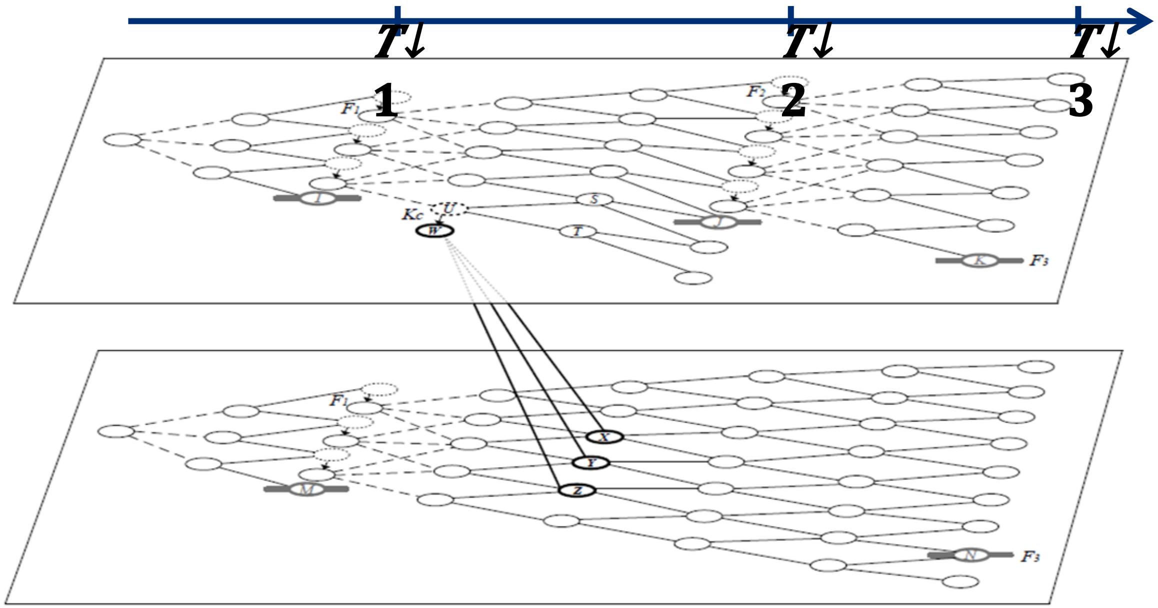




- **To implement the structural model with the debt structure containing callable or puttable bonds that drive premature redemptions, we propose a novel quantitative framework: a forest with multi-layer trees**

  - **To capture the contingent changes of the repayment schedule due to early redemption, more than one tree is needed to make them work collaboratively as a forest. For example, if the aforementioned B_2 is a callable bond, then**

A forest with two-layer tree

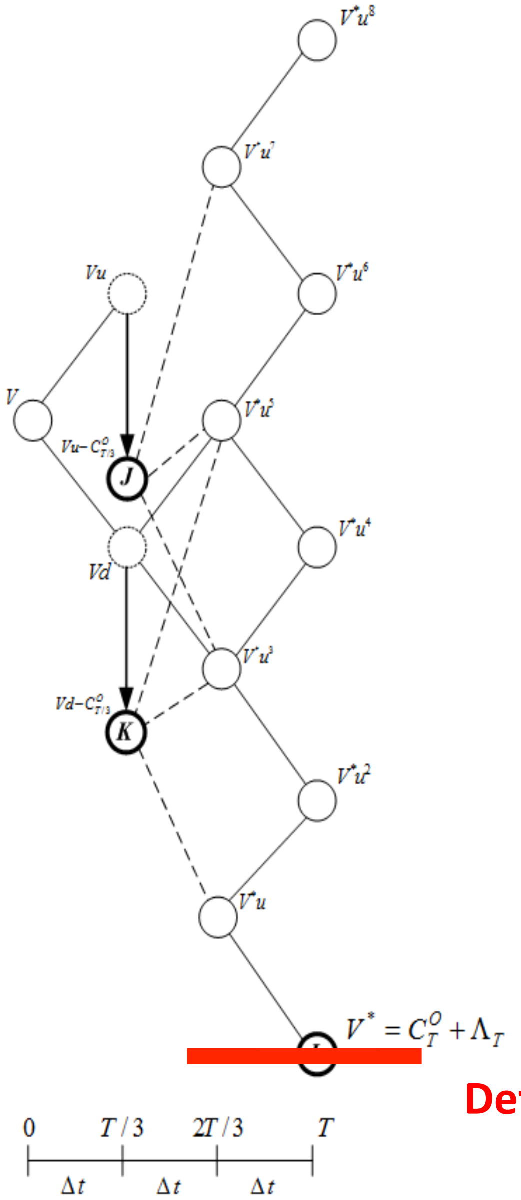


the upper layer tree (before redeeming B_2)

the lower layer tree (after redeeming B_2)

  - **It may be an alternative way to solve the unsolved problem in Jones et al. (1983) (i.e., the problem to evaluate the equity and the corresponding callable bonds of the same firm)**

- **The trinomial structure is used to deal with jumps in a firm's asset value and to coincide the critical locations, such as default boundaries:**



Default Boundary

- With feasible branching probabilities p_u, p_m, p_d that satisfying

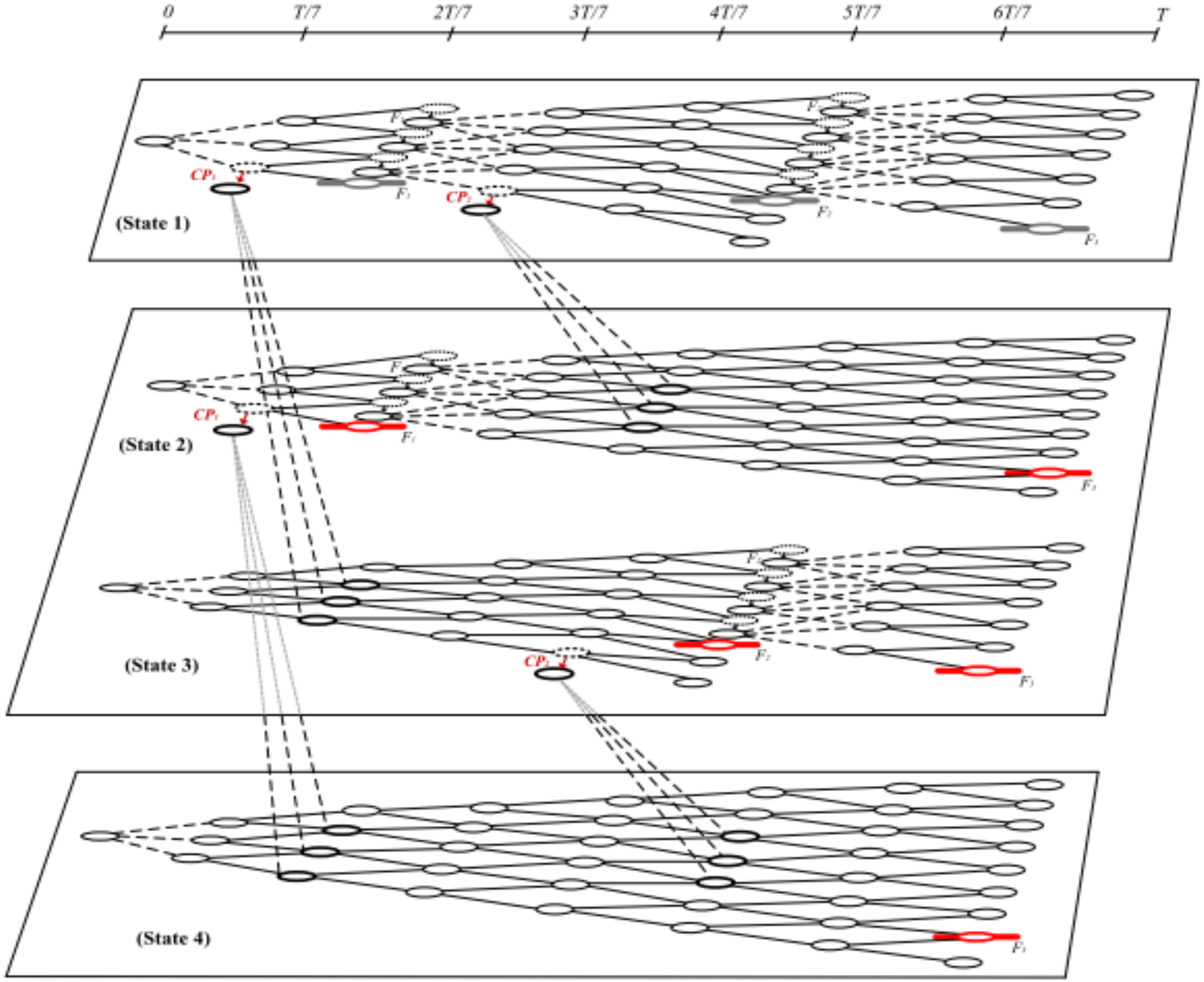
$$\begin{cases} p_u \alpha + p_m \beta + p_d \gamma = 0 \\ p_u (\alpha)^2 + p_d (\gamma)^2 \\ p_u + p_m + p_d = 1 \end{cases} = \sigma^2 \Delta$$

where

$$\alpha \equiv \beta + 2\sigma\sqrt{\Delta t}$$
$$\beta \equiv \mu - \mu$$
$$\gamma \equiv \beta - 2\sigma\sqrt{\Delta t}$$

Dai and Lyuu (2010)

A forest with three-layer trees



- Analyze the optimal call policy under complex debt structure with Forest

[1] Call policies can be measured by (a) the call boundary measured in the firm's asset value;
(b) the premium over effective call price (PoCP)

[2] With the call policy that maximizes the equity value,

[1]-1: low interest rate levels precipitate call (King and Mauer, 2000)

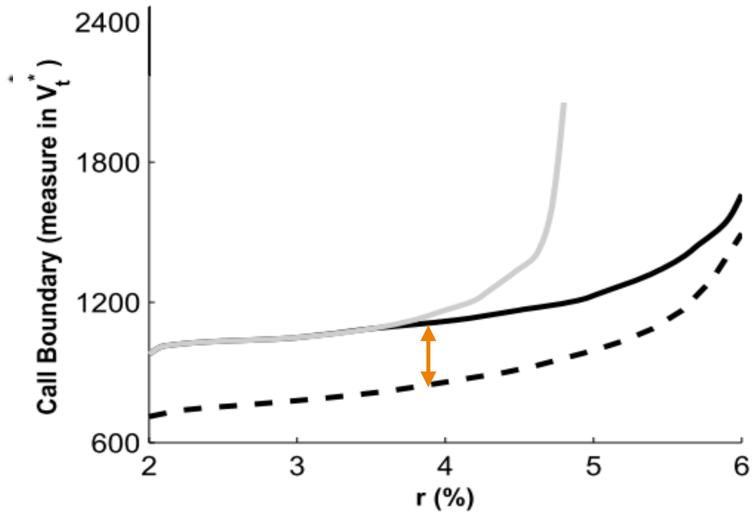
[1]-2: the issuing firm's good credit quality precipitates call

[1]-3: the long-term callable bond will be called before the short-term (King and Mauer, 2000)

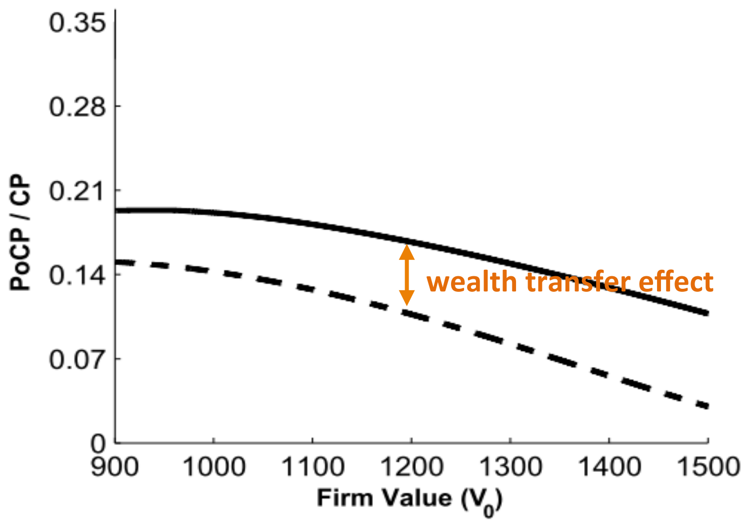
[1]-4: the bond in the debt structure containing more than one bond will be called later than the otherwise identical bond in that containing only one bond (Longstaff and Tuckman, 1994)

Gray solid curve: the policy to call both the long- and short-term bond
Black solid curve: the policy to call only the long-term bond

Solid curve: call policy in the multiple bond case
dashed curve: call policy in the single bond case



(a) given the firm's creditworthiness



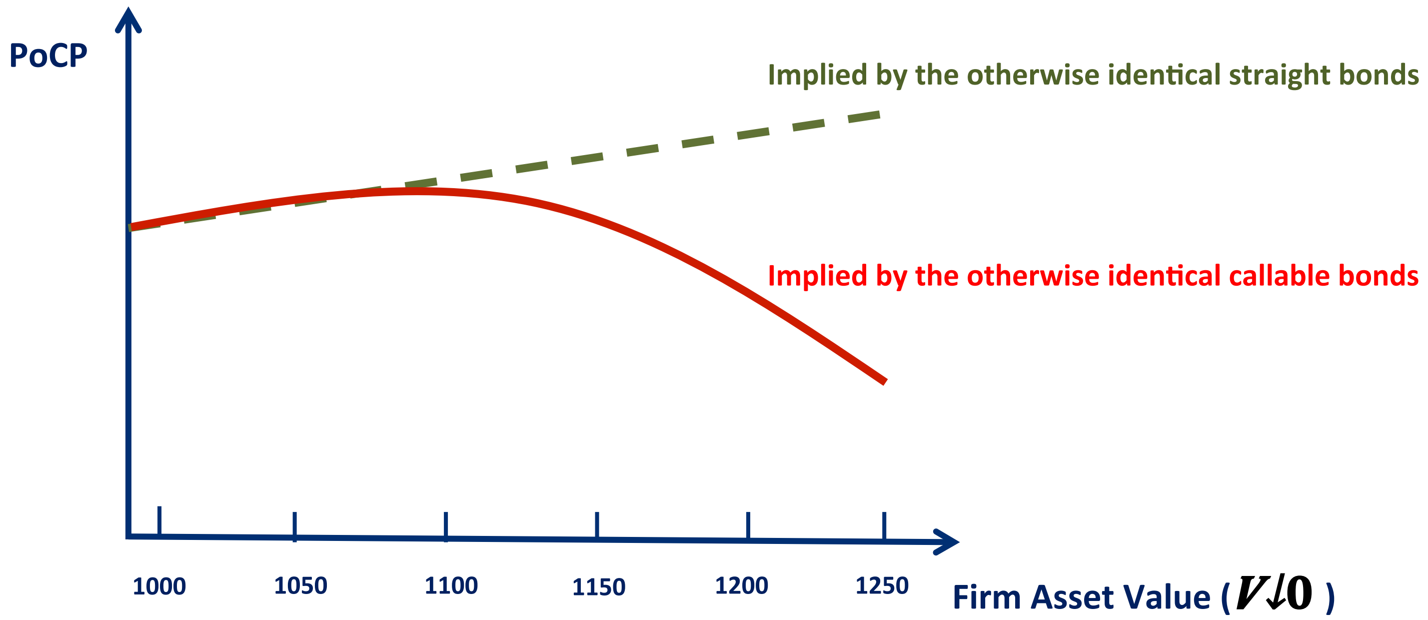
(b) given the level of interest rate

[3] However, regarding the aforementioned right figure, previous literature draws contradicted conclusions from their sample data

- Longstaff and Tuckman (1994) predict the hump-shaped curve
  King and Mauer (2000) observe the upward-sloping curve

- Recall that, other things being equal, a callable bond is more similar to a straight bond when
  [3]-1: the level of interest rate is high
  [3]-2: the issuing firm is unhealthy
  [3]-3: the callable bond is short-term
  [3]-4: the callable bond is in the complex debt structure

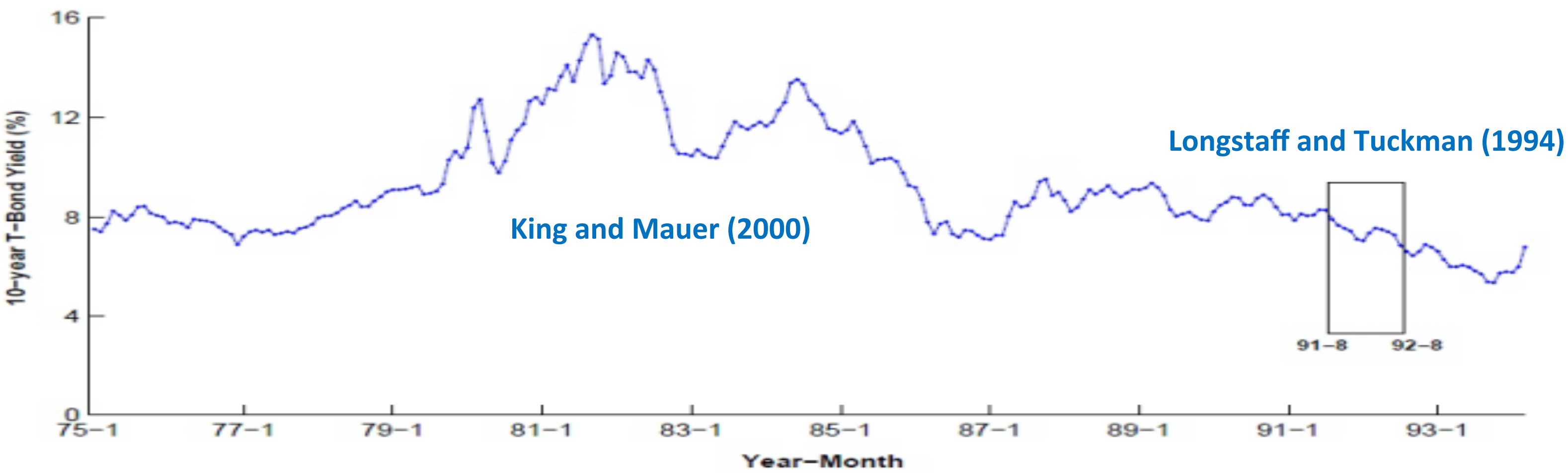
- Premium over Effective Call Price (PoCP) = callable bond price – its effective call price (CP)

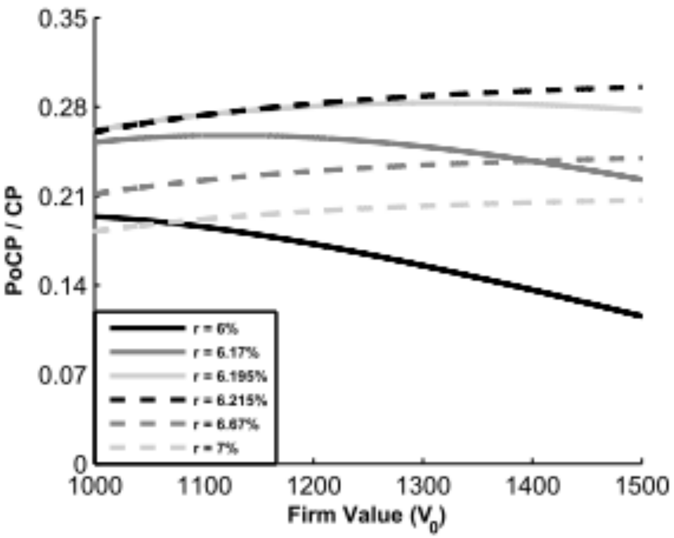


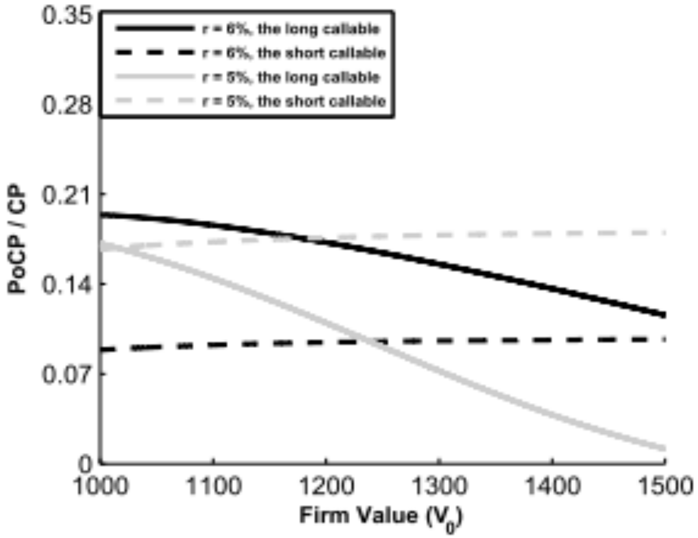
- the conflict may be due to two possible reasons at least

<1> the level of interest rate

<2> simultaneous presence of short- and long-term callable bonds in the debt structure







Conclusions

- **We develop a structural model of credit risk considering the issuing firm's debt structure to simultaneously evaluate its outstanding equity and bonds**
  - **we construct the debt-structure-dependent default trigger that characterizes observable properties of the debt structure**
    e.g., [1] leverage ratio [2] maturity structure [3] priority structure [4] covenant structure
  - **existing stringent assumptions on issuing firm's refinancing policy are released**
  - **the new quantitative method, forest, is proposed to deal with contingent changes of the debt structure due to premature redemptions**
- **Compared with existing structural models, our valuation framework can produce the results more consistent with the observations documented in empirical studies and provide theoretical insights into them**
- **Through this framework, we shed light on the effect of considering corporate debt structure on bond evaluation and the relevant claim holders' decision**

Q&A

Appendix

Appendix A. Model Framework

[1] To meet the investment and finance requirements at different periods of time,

a firm would issue multiple bonds with different maturities, seniorities and covenants at different times. Let the firm's debt structure be composed of *N* outstanding bonds.

[2] The *i-th* bond B_i with face value F_i and time to maturity T_i promises the

annual coupon payment C_i. Let $0 < T_1 < \ldots < T_N = T$.

The firm's debt structure then implies its contractually-obligated payments, including those scheduled and those occurring contingently, like premature redemption.

Denote these time-dependent debt service payments by $\{C_t^o : 0 \leq t \leq T\}$.

[3] **Denote the market value of the firm's asset at time t by V_t**

Under the structural model, these bonds and the corresponding equity are contingent claims on the issuing firm's asset value, denoted by $B_1(t, V_t)$,…, $B_N(t, V_t)$ and $E(t, V_t)$

[4] **The aforementioned time-dependent payments are captured faithfully.**

For example, let $C_{t_j}^O$ and $C_{t_{j+1}}^O$ be two required payments made discretely at
time t_j and t_{j+1}, $t_j < t_{j+1}$; t^- and t^+ denote the time immediate before
and after a required payment.

Then,

$$\begin{cases} V_{t_j^+} = V_{t_j^-} - C_{t_j}^O \\ V_{t_{j+1}^+} = V_{t_{j+1}^-} - C_{t_{j+1}}^O \end{cases},$$

where $V_{t_j^-}$ is the remaining asset at time t_j^- and is utilized to measure firm's solvency at time t_j^-. The effect of repaying $C_{t_j}^O$ on other outstan claims of the same firm is then captured.

[5] The above $V_{t_{j+1}}^{-}$ is derived from $V_{t_j}^{+}$ and the dynamics

$$dV_t = rV_t\,dt + \sigma V_t\,dz,$$

where

[1] r is the long-term average risk-free rate;

[2] σ is the volatility of the firm asset value, representing the firm's business risk;

[3] dz is a standard Brownian motion;

Attaoui and Poncet (2013)

Merton (1974)

[6] The firm determines to file for bankruptcy at t^{-} once $C_t^{O} > 0$ and $V_{t^{-}} < C_t^{O} + \Lambda_t$, where Λ_t is the asset frozen by the undue secured bonds or restrictive covenants, like negative pledge covenants.

Ou et al.(2006) – payment default

That implies $E(t^{-}, V_{t^{-}}) = 0$ when the firm files for bankruptcy.

[7] Once the firm files for bankruptcy, it is liquidated immediately.

The leftover assets are then distributed according to the absolute priority rule.

Bris et al.(2010) – Immediate liquidation

Bris et al.(2006) – No violation of absolute priority rule

[8] When using deb capital, the firm earns tax shield benefits as long as the is solvent but incurs bankruptcy costs when it is insolvent. Leland (1994)

[9] When raising debt capital in the bond market, the issuing firm incurs a proportional cost of k, $k \in (0,1)$, expressed as the market value of the newly issued bond. Chen (2010)

- For example, at time t

  (1) if the firm issues a new bond, B_{N+1}, with the market value $B_{N+1}(t, V_t)$, then

$$V_t^{+} = V_t^{-} + (1-k) B_{N+1}(t, V_t)$$ and $\{C_t^{O} : t^{+} < t \leq T\}$ increases given $T_{N+1} = T$

  (2) if the firm issues the new bond, B_{N+1}, to replace the existing bond B_N, then

$$V_t^{+} = V_t^{-} - B_N(t, V_t) + (1-k) B_{N+1}(t, V_t)$$

- The effect of the decision to increase the firm leverage by issuing new bonds on other outstanding bonds of the same firm can be faithfully captured as the previous empirical literature. (Collin-Dufresne et al.,2001b; Flannery et al, 2012)

- Without changing the total amount of debt, the effect of the decision to rollover the about to mature bonds on other outstanding bonds of the same firm can also be faithfully captured as the previous empirical literature. (Gopalan et al., 2014; Nagler, 2014)

[10] The yield to maturity of the bond B_i at time 0, $Y_0^{B_i}$, can be implied by

$$B_i(0,V_0)=F_i e^{-T_i Y_0^{B_i}}+\sum_{j=1}^{nT_i} C_i/n\, e^{-j/n\, Y}$$

and the corresponding yield spread is

$$Spread_0^{B_i}=Y_0^{B_i}-r$$

Appendix B. Numerical Implementation
-binomial and trinomial tree

- **CRR Trees for the lognormal diffusion process:**

$$dV_t = rV_t\,dt + \sigma V_t\,dz$$

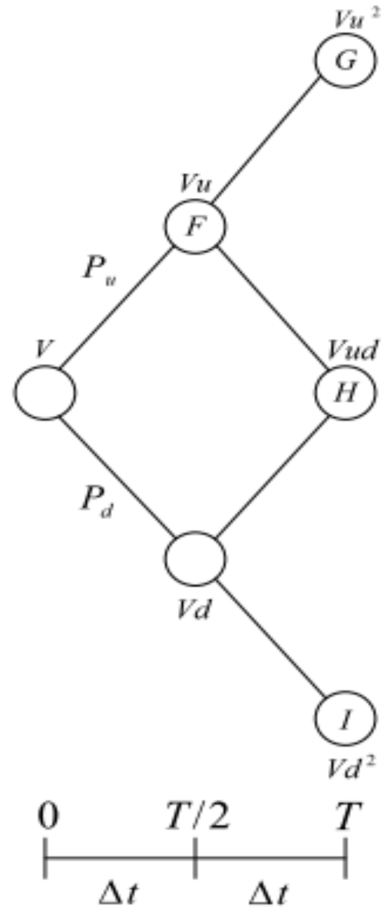
[1] Size of one time step: $\Delta t = T/n$

[2] 4 parameters: u, d, P_u , P_d :

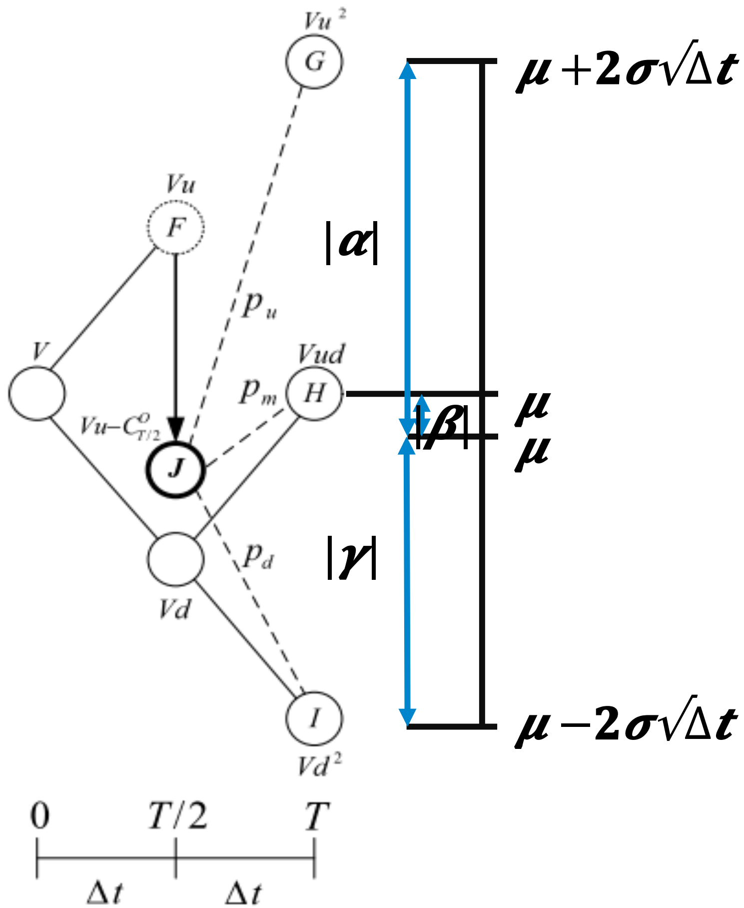
- $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$;

Cox, Ross and Rubinstein (1979)

- $P_u = e^{r\Delta t} - d/$



- **The trinomial structure is used to deal with jumps in a firm's asset value and to coincide the critical locations, such as default boundaries**:



- With feasible branching probabilities p_u, p_m, p_d that satisfying

$$\begin{cases} p_u\alpha+p_m\beta+p_d\gamma=0 \\ p_u(\alpha)^2+p_d(\gamma)^2=\sigma^2\Delta \\ p_u+p_m+p_d=1 \end{cases}$$

where

$\alpha\equiv\beta+2\sigma\sqrt{\Delta t}$

$\beta\equiv\mu-\mu$

$\gamma\equiv\beta-2\sigma\sqrt{\Delta t}$

Dai and Lyuu (2010)

Appendix C. Our Model Produces the Following 8 Sets of Observations Documented in Empirical Studies

[1] upward sloping yield spread curves
(Helwege and Turner, 1999 ; Huang and Huang, 2008)

[2] spread-rate relation
(Duffee, 1998)

[3] spread-volatility relation
(Avramov et al.,2007)

[4] spread-leverage relation
(Collin-Dufresne et al., 2001b; Flannery, 2012)

[5] the effect of rollover risk on existing bonds
(He and Xiong, 2012; Gopalan et al., 2014; Nagler, 2014)

[6] the effect of junior bond issuances to replace bank loans on senior unsecured bonds with or without payment blockage covenants
(Linn and Stock, 2005)

[7] call delay phenomena
(Longstaff and Tuckman, 1994; King and Mauer, 2000; Jacoby et al., 2010)

[8] the effect of including poison put covenants on bidders' cost of debt
(Cook and Easterwood, 1994; Cremers et al., 2007)