A Novel Tree Model for Evaluating Corporate Debts with Complex Liability Structures and Debt Covenants

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Outline

Introduction

- A default-free bond can be priced independently from other outstanding bonds of the same issuer.
- A risky bond, however, cannot be evaluated independently from other outstanding bonds of the same issuer.
- This is because they may change the financial status of the issuer and, as a result, the likelihood of default.

Credit Models

- There are two kinds of credit models: the reduced-form model and the structural model.
- This paper will focus on the structural model.
  - It models the evolution of the firm's asset value and specifies the conditions leading to default.

Structural Credit Models

- Merton (1974)
  - Default can only occur at the bond's maturity date when the firm's asset value cannot meet its payment obligations.
- Black and Cox (1976)
  - The firm defaults once its asset value hits an exogenous default boundary.
- The collection of a firm's outstanding bonds constitutes its liability structure.
- Most of the literature focuses on a liability structure consisting only of a single bond.
- It is difficult to extend their analytical results under more general liability structures.

Bond Covenants

- Bond covenants also affect bond prices.
- Common bond covenants include restrictions on asset sales, exogenous default boundaries, seniorities of bonds, and early redemption.

Assumptions on Asset Sales

- The values of risky bonds strongly depend on the assumptions regarding asset sales.
- The no-asset-sales assumption:[1]
    - The equity holders are not allowed to sell the firm's assets to finance the coupon, bond, or dividend payouts.
    - Thus the equity holders have to finance the payments by issuing new equities.

[1] Leland (1994).

Assumptions on Asset Sales (concluded)

- But allowing asset sales is more common in the real world.
- The proportional-asset-sales assumption:[2]
  - The firm is allowed to sell a proportion of the firm's asset value.
    - If the said proportion of the firm's asset value is less than the payout, the equity holders will try to finance the shortfall by selling additional equities.
- The total-asset-sales assumption:[3]
  - The firm is allowed to finance the total payout by selling the firm's asset.
  - This assumption significantly increases the analytical complexity.

[2] Leland (1994), Kim et al. (1993), Hilberink and Rogers (2002).
[3] Brennan and Schwartz (1978).

Default Events

- The default event is triggered once the firm's asset value hits a default boundary.
- The boundary at time t can be exogenously specified as a function of the firm's outstanding bonds at time t.
  - A constant proportion of the sum of the outstanding bonds' face values.[4]
  - The discounted present value of the outstanding bonds.[5]
- The default boundary can also be endogenously defined.
  - The firm fails to raise sufficient equity capital to meet current bond obligations.[6]

[4] Nielsen et al. (2001), Kim et al. (1993), Longstaff and Schwartz (1995).
[5] Black and Cox (1976), Briys and De Varenne (1997).
[6] Leland (1994).

Pricing Risky Bonds

- Pricing a risky bond in the presence of other outstanding bonds seems to be first studied by Geske (1977).
- Under the no-asset-sales assumption, the equity value and the longest-term bond can be priced as compound options.
    - Geske (1977) assumes that there is no exogenous default boundary.
- However, it is difficult to extend this method under different bond covenants or assumptions on asset sales.

Main Results

- This paper proposes a flexible lattice for pricing risky bonds with general liability structures and bond covenants.
- To model the jumps in the firm's asset value because of the coupon or bond repayment, we adopt the trinomial structure of Dai and Lyuu (2010).
- Our lattice has the flexibility to eliminate the price oscillations by making certain nodes or price levels on the lattice align with the exogenous default boundaries.
- Our lattice can deal with the endogenous default boundary, the early redemption, and seniority of bonds.
- Finally, our lattice can deal with the jump-diffusion process.

The Dynamics of the Firm's Asset Value

- Denote the firm's asset value at time t as V_t.
- The firm's asset value follows the jump-diffusion process.[7]

$$dV_t = \left(\left(r - \lambda\bar{k}\right) V_t - P\right) dt + \sigma V_t\, dz + kV_t dq,$$

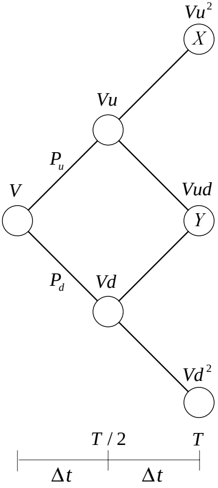
where

- r is the risk-free rate;
- P denotes the payout from selling the firm's asset per annum to finance bond payouts;
- σ denotes the volatility contributed by the diffusion component;
- dz is a standard Brownian motion;
- k denotes the magnitude of the random jump;
- q denotes a Poisson process with an intensity λ.

[7] Zhou (2001).

Lattice Structures under the Lognormal Diffusion Process

- The size of one time step is $\Delta t = T/n$.
- u, d, P_u, P_d:
  - Match the mean and variance of the return asymptotically.
  - $ud = 1$.
  - $P_u + P_d = 1$.
- The trinomial structure is used to deal with the jumps in a firm's asset value.



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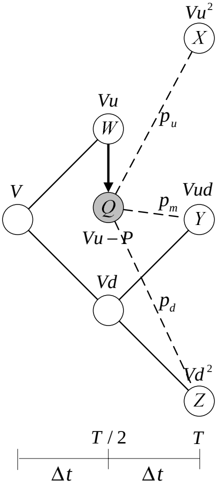
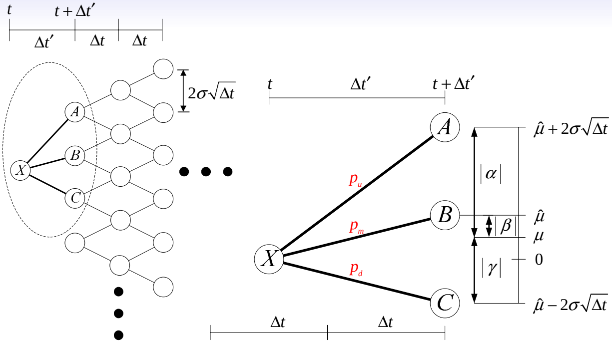


Price Oscillation Problem

- Price oscillation problem is mainly due to the nonlinearity error.
  - Introduced by the nonlinearity of the contingent claim's value function.
- The solution of the nonlinearity error:
  - Making a node or a price level of the lattice coincide with the critical locations where the value function of the contingent claim is highly nonlinear.
- For the structural model, critical locations occur along the exogenous default boundary and at the time points when bond payouts occur.

Trinomial Structure



The branching probabilities for the node X

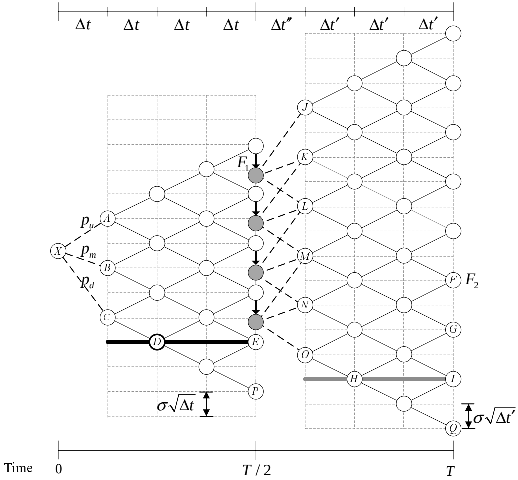
$$P_u\alpha + P_m\beta + P_d\gamma = 0,$$

$$P_u(\alpha)^2 + P_m(\beta)^2 + P_d(\gamma)^2 = \text{Var},$$

$$P_u + P_m + P_d = 1.$$

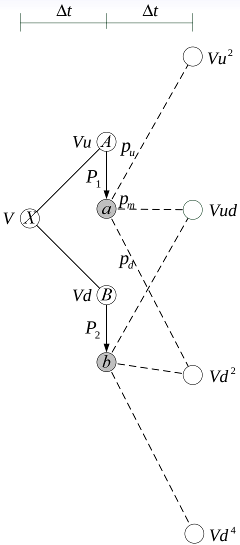
An Example

- Two zero-coupon bonds (face value, maturity date):
  - $(F_1, T/2)$
  - (F_2, T).
- The bond repayments are fully financed by selling the firm's asset.
- The default boundary:
  - $\kappa(F_1 + F_2)$ for $[0, T/2]$.
  - κF_2 for $(T/2, T]$.



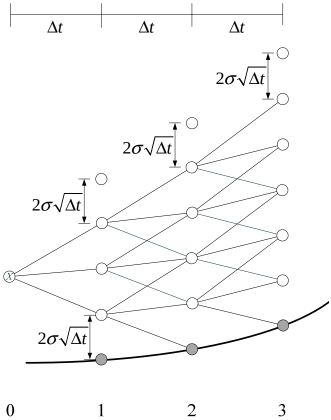
Extensions to Other Bond Covenants

- Incorporate different assumptions regarding asset sales.
- P_1 and P_2 denote the payouts financed by selling the firm's asset.
  - The proportional-asset-sales assumption of selling a fixed proportion D of the firm's asset value:
    - $P_1 \equiv D\, V_u \Delta t$ and $P_2 \equiv D\, V_d \Delta t$.
  - The no-asset-sales assumption:
    - $P_1 \equiv P_2 \equiv 0$.



Extensions to Other Bond Covenants (concluded)

- Assume a time-varying exogenous default boundary.
- The gray nodes are on the curve.
- Other nodes are laid out from the gray nodes upward.
- Thus the successor nodes of node X will be selected from the nodes at time step 1.
- The procedure can be repeatedly applied in the construction of the lattice.



Robustness and Generality

σ_V	Merton (1974) Lattice	Merton (1974) Formula	Black and Cox (1976) Lattice	Black and Cox (1976) Formula
0.25	2934.82	2934.82	2940.03	2940.03
	(0.00003%)		(0.00002%)	
0.4	2875.60	2875.60	2935.53	2935.53
	(0.00014%)		(−0.00010%)	

σ_V	Leland (1994) Lattice	Leland (1994) Formula	Geske (1977) Lattice	Geske (1977) Formula
0.25	3419.57	3419.38	2449.81	2449.79
	(−0.00556%)		(−0.00082%)	
0.4	2941.41	2942.23	2425.37	2425.55
	(−0.02788%)		(−0.00742%)	

Table: Accuracy of Our Lattice.

General Liability Structures

	Prices of B_1			
	$B_2 \prec B_1$		$B_1 \prec B_2$	
Maturity of B_1	Formula	Lattice	Formula	Lattice
2.5	475.59 (M)	475.59	x	366.23
3	470.80 (L)	470.80	342.27 (L)	342.27
3.5	x	466.12	339.06 (G)	339.08

Table: Pricing Unprotected Bonds under the No-Asset-Sales Assumption.

- M: the formula of Merton (1974).
- G: the formula of Geske (1977).
- L: the formula of Lando (2004).

General Liability Structures (concluded)

	The credit spreads (bps) of bond B_1			
	Total-Asset-Sales			
	$B_2 \prec B_1$		$B_1 \prec B_2$	
Maturity of B_1	Non-putable	Putable	Non-putable	Putable
2.99	0.00002	0.00002	1,553.54333	42.35894
3	0.00189	0.00189	1,622.31542	42.51885
3.01	40.86288	0.02893	1,613.55337	42.67799

Table: Impacts of General Liability Structures and Bond Covenants on Protected Bonds.

Conclusions

- This paper prices risky bonds by incorporating general liability structures and bond covenants into the structural model.
- An efficient lattice is then presented to price these bonds.
- This accurate numerical method is of great help to explore how credit spreads are influenced by the bond covenants and the change in the liability structure due to bond repayments.
- Furthermore, our lattice can be extended to deal with jump-diffusion processes.
- The numerical results confirm the robustness and generality of our lattice.
- The also show its ability to accurately evaluate the risky bonds with general liability structures and bond covenants.