An Efficient and Accurate Lattice for Pricing Derivatives under a Jump-Diffusion Process

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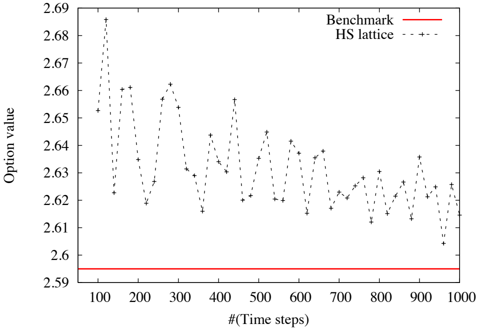
Outline

Introduction

- Pricing derivatives is equivalent to computing its expected payoff under a suitable probability measure.
- Most derivatives have no analytical formulas.
- So they must be priced by numerical methods like the lattice model.

Oscillations

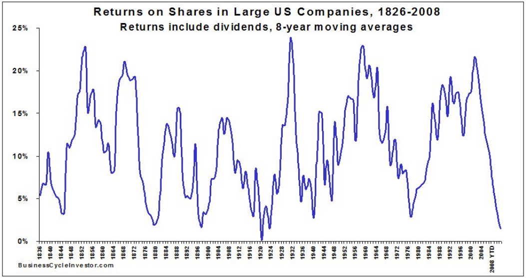
- However, the nonlinearity error may cause the pricing results to converge slowly and oscillate significantly.[1]



[1] Figlewski and Gao (1999).

Models

- Lognormal diffusion process has been widely used to model the stock price dynamics but is incapable of capturing empirical stock price behaviors.[2]
- Many alternative processes like jump-diffusion process have been proposed to address this problem.



[2]Black and Scholes (1973), Hosking, Bonti, and Siegel (2000).

Related Work

- Amin (1993)
  - He approximates the jump-diffusion process by a multinomial lattice.
  - Huge numbers of branches at each node make the lattice inefficient.
- Hilliard and Schwartz (2005)
  - They match the first local moments of the lognormal jumps.
  - Their lattice lacks the flexibility to suit derivatives' specifications.

Main Results

- This talk proposes an efficient lattice model for the jump-diffusion process.
- The time complexity of our lattice is $O(n^{2.5})$.
- Our lattice is adjusted to suit the derivatives' specification so that the price oscillation problem can be significantly suppressed.

Jump-Diffusion Process

- Define S_t as the stock price at time t.
- Merton's jump-diffusion model assumes that the stock price process can be expressed as

$$S_t = S_0 e^{(r-\lambda\bar{k}-0.5\sigma^2)t+\sigma z(t)} U(n(t))). \tag{1}$$

- Decomposing Eq. (1) into the diffusion component and the jump component:

$$V_t \equiv \ln\left(S_t/S_0\right) = X_t + Y_t,$$

  - The diffusion component

$$X_t \equiv \left(r - \lambda\bar{k} - 0.5\sigma^2\right) t + \sigma z(t)$$

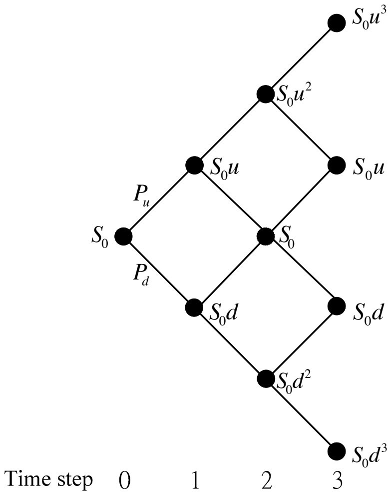
    is a Brownian motion.
  - The jump component

$$Y_t \equiv \sum_{i=0}^{n(t)} \ln\left(1 + k_i\right)$$

    is normal under Poisson compounding.

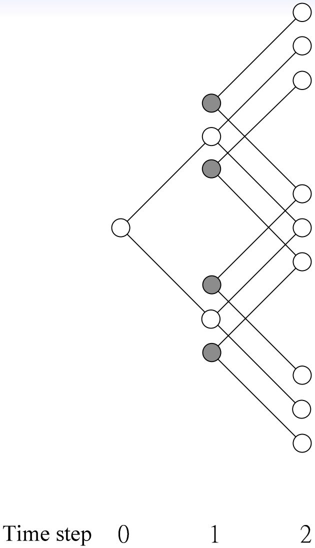
CRR Lattice for the Diffusion Part

- The size of one time step is $\Delta t = T/n$.
- u, d, P_u, P_d:
  - Match the mean and variance of the stock return.
  - $ud = 1$.
  - $P_u + P_d = 1$.



Hilliard and Schwartz's Lattice

- Diffusion part (X_t)
  - Match mean and variance of $X_{\Delta t}$.
  - Obtain P_u and P_d.
- Jump part (Y_t)
  - Match the first $2m$ local moments of $Y_{\Delta t}$.
  - Obtain q_j ($j = 0, \pm 1, \pm 2, \ldots, \pm m$).
  - The node count of the lattice is $O(n^3)$.



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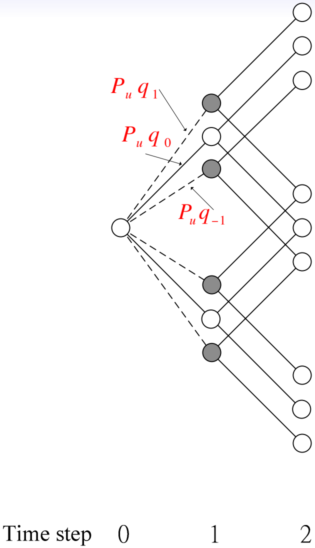
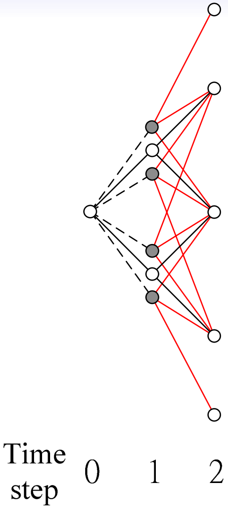
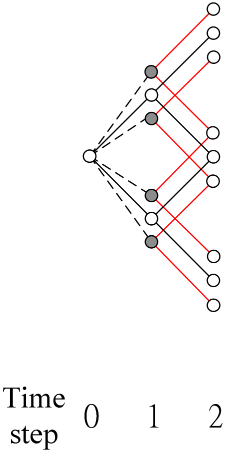


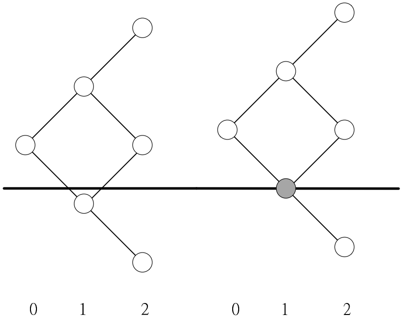
Hilliard and Schwartz's and Our Lattice



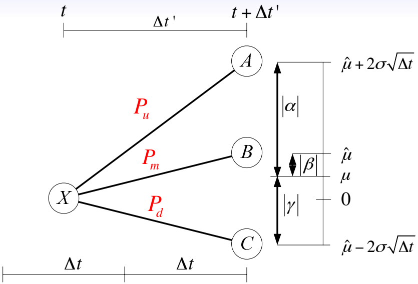


Price Oscillation Problem

- Price oscillation problem is mainly due to the nonlinearity error.
- The solution of the nonlinearity error:
  - Making price level of the lattice coincide with the location where the option value function is highly nonlinear, such as the barriers and strike price.



Trinomial Structure

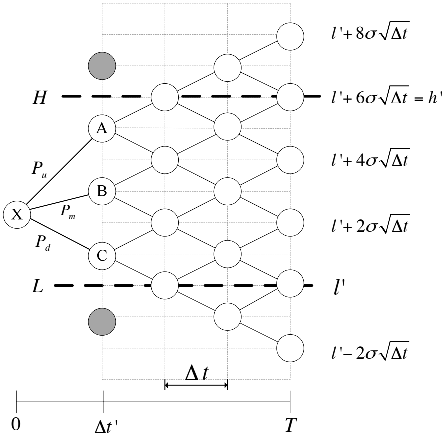


Theorem 1: the branching probabilities for the node X

$$P_u\alpha + P_m\beta + P_d\gamma = 0,$$
$$P_u(\alpha)^2 + P_m(\beta)^2 + P_d(\gamma)^2 = \text{Var},$$
$$P_u + P_m + P_d = 1.$$

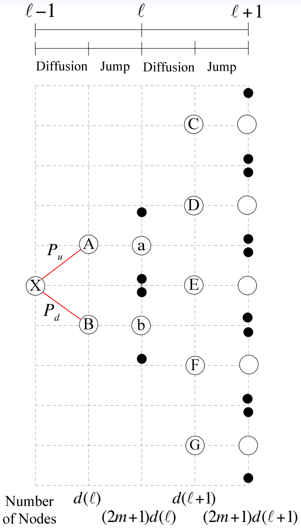
Adjusting the Diffusion Part of the Lattice

- Select Δt to make $\frac{h'-l'}{2\sigma\sqrt{\Delta t}}$ be an integer.
- Lay out the grid from barrier L upward.
- Automatically, barrier H coincides with one level of nodes.
- Obtain P_u, P_m, P_d by Theorem 1 (p. 13).



Dealing with Jump Nodes

- Two phases: the diffusion phase and the jump phase.
- The node count of our lattice is $O(n^{2.5})$.



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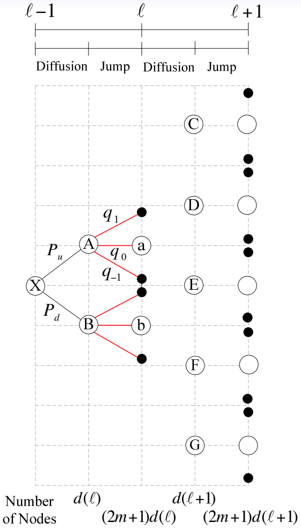

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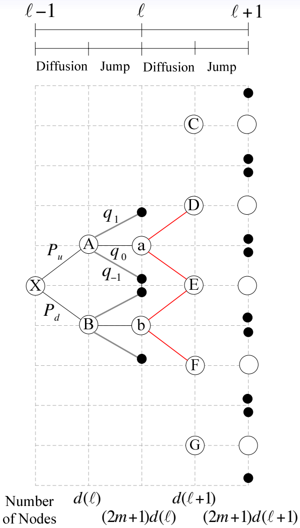

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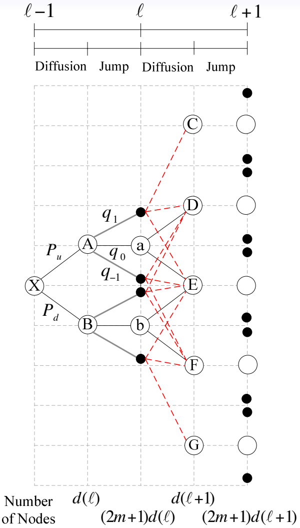

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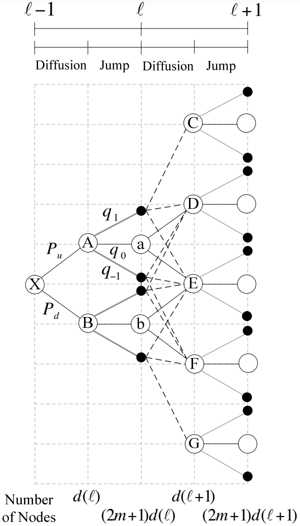

Time Complexity

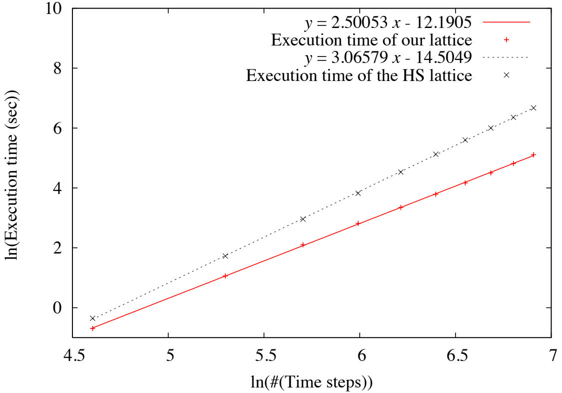

Figure: time complexity.

Vanilla Options

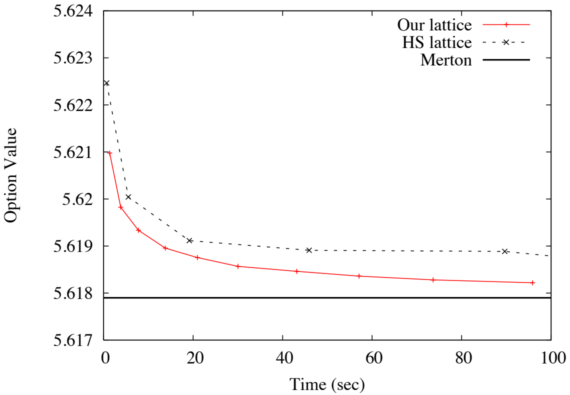

Figure: Converge Property.

Barrier Options

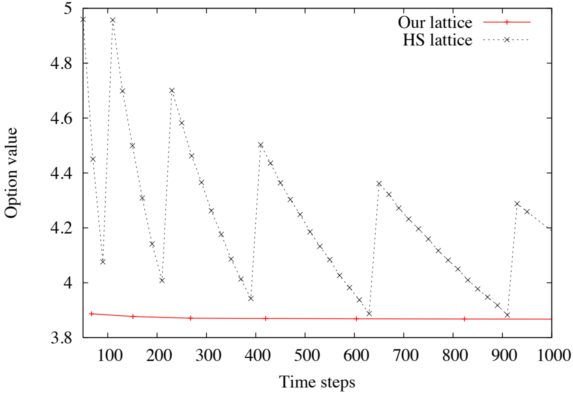

Figure: Pricing a Single-Barrier Call Option.

Conclusion

- This talk presents a novel, accurate, and efficient lattice model to price a huge variety of derivatives whose underlying asset follows the jump-diffusion process.
  - It is the first attempt to reduce the time complexity of the lattice model for the jump-diffusion process to $O(n^{2.5})$.
  - In contrast, that of previous work is $O(n^3)$.
  - With the adjustable structure to fit derivatives' specifications, our lattice model make the pricing results converge smoothly.
- According to the numerical results, our lattice model is superior to the existing methods in terms of accuracy, speed, and generality.